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THE POSSIBILITY OF A REALIZATION OF FOUR-FOLD SPACE.[1]

Any magnitude that is a function of a single variable may be represented geometrically by a straight line. Functions of two variables are represented by curved lines or by plane areas; and functions of three variables by either twisted curves, curved surfaces, or volumes. The conceptions of length, area, and volume when used in this way are evidently independent of any of the properties of matter except extension. The question now before us is this, Can we develop and use in a similar way a space-concept which can fully represent a function of four independent variables?

Perhaps most of us can remember times in the course of our education when new conceptions of quantity entered into our conscious life, conceptions which correspond in a general way with those of length, area, and volume, in that they enable us to find at once such relationships as are most frequently required for practical purposes by a general, synthetic, instinctive method. A medical student, instead of memorizing the exact amount of each dose under all possible conditions of the patient, fixes in his mind as in a frame work the medicinal outline of each drug. The student of chemistry does something similar with the elements; the architect has such a concept of structural beauty; the hunter, of the most likely place for game. The sense of propriety, the sense of honor, and numberless other "inbred" or "instinctive" concepts are examples of this mental tendency. There is therefore nothing inherently absurd or improbable in the supposition that any of us may attain to a conception of four-fold space, "as clear as the designer and the draughtsman have of three-fold space."[2] Such a conception would be of great value to all classes of scientists. The biologist could set in this four-fold framework a complete picture of genetic or race relationships; the theologian could use it for the world of spirits; the physicist for forces, etc. By this means ordinary men may become able to see and to develop easily new truths, such as are now revealed only to men of genius and inspiration.

It may be objected that our conception of three-fold space is derived directly from sensations in three-fold space, and that the conception of four-fold space cannot be derived in a similar way, nor yet from sensations in three-fold space. But it is evident that from any sense, from sight, for instance, we get at most a two-dimentional sensation, and it is only by the kind of changes that occur in the sensation that we can infer that a given retinal picture represents extension in two or in three dimensions. In other words, granting, for the sake of the argument, that in sight we perceive directly the existence of two dimensions, it is clear that the existence of a third dimension is solely a matter of inference. It is the simplest hypothesis we can get to explain our sensations. It is conceivable that the hypothesis of a fourth dimension, if it could be made as real to us, might be found of nearly equal value in the simplification of ordinary phenomena. This would be the case if ordinary phenomena involve motion in four independent directions, or if some of the relations of things in the universe, relations not in space, are capable of complete representation in four-fold space. But before we can decide whether or not space and objects of four dimensions exist we must have our ideas of four-fold space developed sufficiently to know what sensations, what visible and tangible phenomena, would be obtained from objects of four dimensions. Up to this time discussions on the reality of four-fold space have been (necessarily) characterized by the absence of evidence for or against.

To develop a clear conception of four-fold space only one course seems to be open, namely, the synthetical study of four-fold geometric figures in the same way that we now study geometric solids. Having given the number and form of the boundaries of a solid we can, by the process of visualization, find more or less easily its appearance (plane projection) in various positions, the possible plane sections, the distance between any two of its points, and so on. In the study of a tessaract (four-fold figure) we should deal similarly with its solid boundaries, finding the possible solid sections, solid projections, and so forth, studying the tessaract by means of conceptions already familiar (length, area, volume), but in new relations. It this way may be developed gradually such a knowledge of the properties of tessaracts as will enable us to "see" them clearly, and to comprehend quickly a new shape. Models of the solid projections and sections are indispensible to rapid progress. Difficulties may, in general, be overcome by considering the analogous difficulties an imaginary plane being, that is to say, a being who has no conception of volume, would have in trying to understand a geometric solid.

The First Lesson.

A point moving in one direction traces a straight line. A line moving perpendicular to itself, in one plane, traces a square; and a square moving similarly traces a cube. How could a plane being learn the number and relations of the faces of a cube? He could readily understand that as the square moves in a direction perpendicular to all of its sides each side traces a new square, and that the moving square in its first and last positions forms the remaining pair of opposite faces. In this way he could count up the six faces, twelve edges, and eight corners of the cube, and might pro-

[1] Digest of a paper read before the Canadian Club of Clark University by T. Proctor Hall, Ph.D

[2] "A New E a of Thought," by C. H. Hinton, M.A.